Serial Concatenated Trellis Coded Modulation with Iterative Decoding[1]

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Abstract — **We** propose a design approach for serial concatenation of an outer convolutional code and an inner trellis code with multilevel amplitude/phase modulations **using a bit-by-bit iterative decoding scheme. An example is given for throughput of 2 bits/sec/Hz with 2x8PSK modulation to clarify the approach. In this example, an 8-state outer code with rate 4/5 and a 2-state inner trellis code with 5 inputs and 2x8PSK outputs per trellis branch were used. The performance of this code with input** block of 16384 bits is within **1.1dB from the Shannon limit for 8PSK at a bit error probability of 5×10^{-8} for 2 bits/sec/Hz with 10 iterations.**

I. INTRODUCTION

Trellis coded modulation (TCM) proposed by Ungerboeck in 1982 [I] is now a well-established technique in digital communications. Turbo codes represent a more recent development in the coding research field [2]. In [3] we merged **TCM** and parallel concatenated codes in order to obtain large coding gains and high bandwidth efficiency. Here we suggest to merge **TCM** with recently discovered serial concatenated codes [4], adapting the concept of iterative decoding used in parallel concatenated codes.

11. SERIAL CONCATENATED TCM

The basic structure of serial concatenated trellis coded modulation is shown in Fig. 1a.

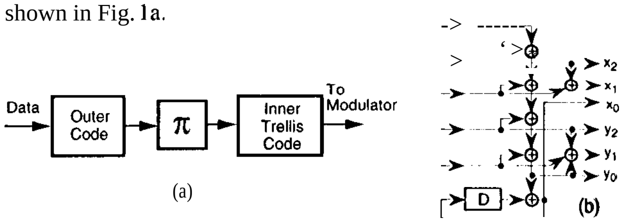

Figure 1: (a) - Serial TCM; (b) - 2-state inner trellis code.

We propose a solution to serial concatenated TCM, which achieves b bits/see/Hz, using a rate $2b/(2b+1)$ non-recursive binary convolutional code with maximum free Hamming distance as an outer code. We interleave the output of the outer code with a random permutation. The interleaved data enters a rate $(2b+1)/(2b+2)$ recursive convolutional code as inner code. We map the $2b+2$ output bits to two 2^{b+1} level modulation (four dimensional modulation).

Design of serial Concatenated **TCM.** Let z bc the binary output sequence of an outer code, and x(z) be the corresponding inner **TCM** encoder output with M-ary symbols. We design the constituent inner TCM encoder such that the minimum Euclidean distance c/(.x(z), $x(z')$) over all z, z' pairs, z # z', is maximized, given that the Hamming distance d_H (z, z') = 2. We call this minimum Euclidean distance the *effective free Euclidean distance* of the inner TCM code and denote it simply by $d_{f,eff}$. If the free distance of the outer code d_f^o is odd, then among the selected inner TCM codes choose those that have maximum Euclidean distance $d(x(z), x(z'))$ over all z, z' pairs, z # z', given that $d_H(z, z') = 3$. We denote this minimum Euclidean distance of the inner TCM code due to input Hamming distance 3 by $h_m^{(3)}$. It can be shown that the dominant term in the transfer function bound on bit error probability of serial concatenated **TCM,** averaged over all possible interleaves of size N bits **(for large N), is proportional** to $N^{-d_f^o/2}e^{-(E_s/4N_o)d_f^o d_{f,eff}^2/2}$ for d_f^o even, and to $N^{-(d_f^o+1)/2}e^{-(E_s/4N_o)((d_f^o-3)d_{f,eff}^2/2+h_m^{2(3)})}$ for d_f^o odd where E_s/N_o is M-ary symbol signal-to-noise ratio. We use well known set partitioning techniques for multidimensional signal sets, using proper input labels assignment based on the codewords of the parity check code $(2b+1, 2b, 2)$ to maximize the quantities described above. For clarification wc use the following example.

Example: Set partitioning of 2 x $8PSK$ **and input labels assignment.** Let the eight phases of 8PSK bc denoted by (O, 1,2,3,4,5,6, 7]. Consider the $2\times8PSK$ signal set A_0 = [(0, O), (O, 4), (2, 2), (2, 6), (6, 2), (6, 6), (4, O), (4, 4)] and the following sets constructed from A_0 as, $A_1 = A_0 + (0, 2)$, $B_0 = A_0 \cup A_1$, $B_1 = B_0 + (0, 1)$, $B_2 = B_0 + (1, 1)$, $B_3 = B_0 + (1, 0)$, where addition is component-wise modulo 8. For unit radius 8PSK constellation, the intra square Euclidean distance of each set B_i is 2. The inter square Euclidean distances $d^2(B_0, B_2) = d^2(B_1, B_3) = 1.17$. The other interdistances are 0.586. Select the input label set L_0 as codewords of the (5, 4, 2) parity check code. Assign these codewords to sets B_0, and B_3. input label set L_1 is assigned to B_1, and B_2, where $L_1 = L_0$ + (O, 0,0,0, 1) modulo 2. This guarantees that the minimum Hamming distances of input labels for each set B_i is 2. A sufficient condition to have very large output Euclidean distance for input Hamming distance 1, is that all input labels to each state bc distinct. Assign $(L_0, B_0), (L_1, B_1), (L_1, B_2), (L_0, B_3)$ to the parallel edges of a 2-state or 4-state trellis structure such that $d_{f,eff}^2$ is maximum and if possible $h_m^{(3)} = \infty$. For 8 and 16-state trellis, we have to partition B_i into A_0 and A_1 to obtain larger intra and inter distances, and input label L_0 into L_{00} (codewords of L_0 with MSB=0) and L_{11} (codewords of L_0 with MSB=1), with similar partitioning for the other input label and remaining signal sets. Having determined the code by its input labels and 4-dimensional output signals, the encoder structure is then defined by selecting any appropriate labels for the 4-dimensional output signals (Wc used natural mapping). The implementation of the 2-state inner trellis code is shown in Fig. 1(b). For this 2-state inner code, $d_{f,eff}^2 = 1.76$, $h_m^{(3)}$ = co. The outer code is an 8-state, rate 4/5, convolutional code with $d_f^o=3$. Since $h_m^{(3)}$ = co then d_f^o is increased to 4.

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1 This research was carried out at the Jet Propulsion Laboratory and at the Politecnico of Torino, under a contract with NASA and under NATO grant CRG951208.